

# Design and Deployment of a Satellite Constellation Using Collaborative Optimization

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**A study of collaborative optimization as a systematic, multivariable, multidisciplinary method for the conceptual design of satellite constellations is presented. Collaborative optimization was selected because it is well suited to a team-oriented environment, such as often found in the constellation design process. The method provides extensive and formal exploration of the multidisciplinary design space and a scalable formulation of the problem without compromising its subsystems' flexibility or eliminating opportunities for collaboration. The feasibility and benefits of the collaborative optimization architecture are highlighted by the successful convergence of an application notional problem to design and deploy elements of a space-based infrared system that provides early missile warning. Furthermore, this study contributes to the existing knowledge of the collaborative optimization method by verifying the feasibility of nongradient optimization algorithms as both system and subsystem optimizers within the architecture. Finally, the demonstrated convergence of this problem, which involves integer variables, also demonstrates the flexibility of the architecture for handling mixed-discrete nonlinear multidisciplinary problems.**

## Nomenclature

$F(z)$	=	system-level objective function (total cost through deployment)
$i$	=	subspace number index
$J_i$	=	subspace $i$ objective function, measures subspace incompatibility with targets
$k$	=	top-level iteration index in penalty method (system-level optimization)
$N$	=	total number of subspaces
$Q_k(z^d)$	=	penalty for noninteger values of discrete variables in penalty method
$r_k$	=	scalar multiplier for subspace objective functions in penalty method
$s_k$	=	scalar multiplier for discrete variable penalty in penalty method
$y_i$	=	duplicates of interdisciplinary variables $z_i$ , used as independent variables in subspace $i$
$z$	=	vector of system-level design variables, which includes all interdisciplinary variables
$z^d$	=	vector of discrete members of system-level design variables
$z_i$	=	subvector of $z$ provided to subspace $i$ to be used as compatibility target parameters
$\Phi_k(z)$	=	system-level pseudo-objective function, which includes penalty terms
$\psi_i$	=	vector of independent variables in subspace $i$ that are not interdisciplinary variables

## Introduction

**T**HE advantages of satellite constellations have long been appreciated, principally for their coverage capabilities. Many mis-

sions have global observation requirements that simply cannot be met by single-spacecraft systems. Other benefits include increased mission robustness and survivability.<sup>1</sup> Multisatellite systems can tolerate limited failures in individual spacecraft in the constellation and still accomplish all or most of the mission objectives.

Recently, there has been a surge in the number of space architecture concepts proposing to utilize multisatellite constellations to meet demanding mission requirements. This growing emergence of constellation concepts, coupled with the high development cost these systems entail, prompts the need for a systematic optimization approach to perform constellation design. In addition to traditional performance and coverage parameters, the new optimization approach must consider disciplines such as manufacturing and deployment costs, as necessitated by the increased emphasis on cost or profitability for space systems.<sup>1–3</sup> The key to creating a cost-viable satellite constellation, and, therefore, the focus of this research, begins in the early phases of the system design.

Concept development is the first of a sequence of steps that transforms ideas into products. Within this initial design phase, mission objectives and requirements are more concretely defined, and several possible alternative solutions are generated. These candidate solutions are then explored in greater detail, their feasibility assessed, and the preferred concept is selected before the project can progress to the next stage of its design cycle. As a result, the decisions made during the initial phases of design have major impacts on the final product and its cost.<sup>2</sup>

To aid engineers in this decision making, analysis tools and methodologies that are appropriate to the conceptual design environment are needed. Because of often-compressed design schedules, the tools must be capable of rapid turn-around analyses without compromising accuracy. The methodologies must enable engineers to explore the design space efficiently and help them make smart decisions quickly.

The conceptual design of a satellite constellation, however, is a complex, highly constrained, multidisciplinary problem. A graphical representation of the various factors and disciplines that have major influence in the design of a satellite constellation is given in Fig. 1, created from expert sources and work presented in existing literature.<sup>4–6</sup> Typical interdisciplinary variables that couple the disciplines together are shown in Fig. 1. The mutual dependence of one discipline on another results in an iterative problem. The satellite constellation problem also entails a mix of discrete and continuous parameters and variables, which leads to increased optimization difficulties. The inherent nonlinearities of the problem further add to the complexity. Several of the constraints are nonsmooth and/or discontinuous, for example, integer limits on launch vehicle flight rates.

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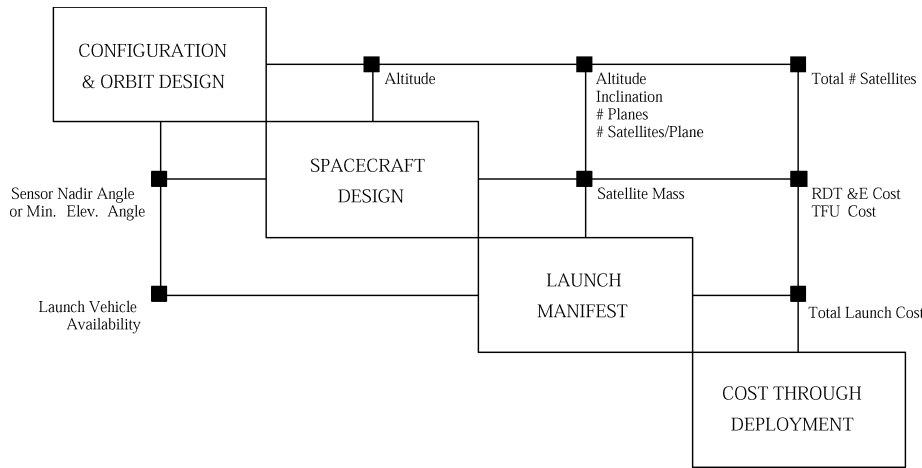


Fig. 1 Satellite constellation design problem.

Table 1 Trade issues for satellite constellation designs<sup>a</sup>

Variable	Configuration and orbit design	Spacecraft design	Launch manifest
Altitude	↑	↓	↓
Inclination	↑	—	↓
Minimum elevation angle	↓	↑	—
Number of planes	↑	—	↓
Number of satellites per plane	↑	—	↓

<sup>a</sup>Arrows signify directions of improved subsystem performance.

Finally, the disciplines themselves have traditionally been optimized independently.

Traditional constellation design variables, such as the altitude of satellite deployment, can easily be claimed by more than one discipline, which leads to difficulties for system integration and overall system optimization. As shown in Table 1, the direction of improved subsystem performance for a given shared design variable might be in conflict from one discipline to another. That is, certain design variables produce opposing effects in the different disciplines, and the final values will affect the overall evaluation criterion, that is, objective function. To maintain internal data compatibility, only one value of altitude or number of orbital planes may be selected, but which discipline should be granted this responsibility? Clearly, some formal method to produce an optimum compromise is required. A systematic, multivariable, multidisciplinary method that can handle these complexities can prove to be beneficial.

### Proposed Method

Multidisciplinary design optimization (MDO) was established as a field of aerospace research in response to the need for a systematic methodology in designing complex systems.<sup>2</sup> MDO encompasses many new integration and optimization techniques that have been proven on various complex multidisciplinary design problems.<sup>7</sup> The thesis of this study is that MDO will prove to be beneficial when applied to the design and deployment of a satellite constellation.

Numerical optimization has been shown to improve the design and deployment of a spacecraft in several previous studies.<sup>3,8</sup> Its application to the orbital parameters of a given constellation design has produced improved configurations for various missions with different objectives.<sup>9–11</sup> Optimization has further been applied to the constellation design at the system level, accounting for payload and spacecraft bus sizing/selection and deployment strategies.<sup>1,12</sup>

The contributions of MDO, however, lie not only in the application of optimization techniques. MDO methods can also address existing organizational challenges<sup>3,13</sup> by providing efficient team-oriented, collaborative decomposition approaches that are suitable for the distributed, multidisciplinary design groups often found working on large space projects today. A distributed analysis architecture is one that decomposes the overall system synthesis prob-

lem into smaller, usually discipline-oriented, tasks, coordinated at the system level.<sup>7,13–15</sup> The organizational and computational advantages of this approach for large-scale complex design problems<sup>7</sup> resulted in its selection as the constellation design architecture to be investigated.

The distributed analysis architecture naturally fits into the multidisciplinary analysis environment of most constellation design groups, where specialists are organized by their area of engineering expertise and where interdisciplinary interactions are fairly complex. It provides the subsystems with independence with regards to selection of computing platforms and preferred optimization algorithms. The independence provided to the disciplines further allows modularity because each subsystem is free to modify its own analysis method, model, assumptions, etc., without requiring the restatement of the overall problem or alterations in the other disciplines typical in a single, monolithic optimization approach. Finally, this modularity provides opportunities for parallel computations. Concurrent processing of the subsystems' analyses, perhaps even on heterogeneous platforms from remote sites, can ultimately lead to reduced design cycle times.

Several methodologies have been developed in the past decade to utilize a distributed architecture for solving large-scale problems, some of which were motivated by the desire to provide the individual disciplinary experts with the most flexibility possible.<sup>7,15</sup> One of these new methodologies is collaborative optimization (CO) proposed by Braun.<sup>13</sup> Unlike an all-at-once approach combining all independent variables and constraints from a multidisciplinary problem into a single monolithic optimization problem, CO employs a bilevel optimization technique wherein a single system optimizer orchestrates and coordinates several optimization processes at the subspace level, that is, at the individual discipline or subsystem level. This bilevel approach provides the subsystems with opportunities to contribute to the overall design decision process by giving them control over local variables and responsibility for satisfying local constraints, while maintaining management of the interdisciplinary compatibilities and overall system-level objective at the top level of the process.

CO has successfully been applied to large-scale multidisciplinary optimization problems related to aircraft and space vehicle design.<sup>13,16,17</sup> The present study's authors suggest that the constellation design problem can similarly benefit from CO implementation. At the same time, the present study will test the feasibility of CO for solving a mixed-integer nonlinear multidisciplinary problem with nongradient-based optimization techniques.

For readers unfamiliar with the CO, a brief introduction is provided. Within CO, the system-level variable vector  $z$  consists of any top-level design parameters that affect the system objective, as well as all interdisciplinary variables that couple the  $N$  subspaces. The system-level optimizer's task is to select values for  $z$  such that the overall objective function  $F(z)$  is minimized (or maximized). However, by selecting  $z$ , the system-level optimizer also sets values

for the interdisciplinary variables (local inputs and outputs passed between disciplines) and poses them as targets for each of the individual subsystem optimizers to match simultaneously within a given system-level iteration. How well these targets are met by each of the subsystems, in turn, become the compatibility constraints,  $J_i$ ,  $i = 1, \dots, N$ , that must ultimately be satisfied at the system level.

The formulation of  $J_i$  is as follows. Assume that  $z_i$  is an  $n_i$ -element subvector of  $z$  composed of all variables that are normally input to or output from subspace  $i$ . Each subspace also has its own local version  $y_i$  of  $z_i$ , as well as noninterdisciplinary local variables  $\psi_i$ . The task of the subspace optimizer in a given system-level iteration of CO is to vary the values of  $\psi_i$  and that portion of  $y_i$  that corresponds to its traditional disciplinary inputs such that the entire vector  $y_i$  (consisting of the inputs and outputs of subspace  $i$ ) is as close as possible to the system-level optimizer's target  $z_i$ . This is achieved by formulation of the objective function for each subspace as  $J_i = \|z_i - y_i\|^2$ .

The subspace optimizers must additionally always ensure that their local disciplinary constraints are satisfied at each system-level iteration, leading to situations in which  $J_i$  might not be zero for a given  $z_i$ . For example, the system optimizer might set a target for a coupling variable that a particular subspace cannot meet while still satisfying its local constraints. Consequently, the system-level optimizer must find  $z$  such that the compatibility constraints are satisfied, that is,  $J_i = 0, \forall i$ , at the final converged solution for minimum  $F(z)$  because when  $J_i = 0, \forall i$ , then the interdisciplinary variable values used by each subspace are feasible.

This division of tasks from the overall problem is a feature of CO that can facilitate the solution-finding process and increase the scalability of the method. These advantages are due mainly to the typical reduction in the number of variables and constraints involved in the system optimization problem. The subsystems can be trusted with local decision parameters  $\psi_i$  and local constraints that do not explicitly affect the system objective or the evaluation of the other subsystems. Therefore, the growth of problem complexity is slowed, and communication requirements between the system and the subspace analyses are easier with smaller demands on data exchange (only  $z_i$ ). These characteristics become important as disciplinary fidelity or breadth is added to the problem, which would result in greater numbers of design variables and constraints at the discipline level.

There are limits to these benefits, however. Problem sparseness is crucial to obtain these savings in terms of computational costs. That is, in the worst-case scenario, where every variable affects every disciplinary analysis ( $z_i = z, \forall i$ ), CO will not provide a significant advantage over methods such as the all-at-once approach because each subspace would have to consider all of the system design variables. Thus, having comparatively few interdisciplinary couplings, as is the case for the constellation design problem, is key to CO's utility. Furthermore, the combined computational effort of the decomposed tasks in CO, even with the concurrent operations, can be fairly intensive because the subsystems are required to perform local optimization at each iteration, rather than the much simpler role of function evaluations that may be required by other methods.

### Application Example

The space tracking and surveillance system (STSS) is the risk-reduction program utilized by the U.S. Missile Defense Agency to develop a system of space-based sensors for the detection, tracking, and discrimination of threat missiles. By the use of an incremental approach, the goal of the program is to develop, through a series of proof-of-concepts demonstrating increasing capabilities, a low-Earth orbit (LEO) constellation that can accurately acquire a missile's state vector data throughout the different phases of its trajectory. The precision tracking capability provided by this LEO system is critical for an effective ballistic missile defense.

The current STSS constellation configuration design is still not definitive.<sup>18,19</sup> To attain continuous global coverage, 20–30 satellites launched by several Delta IIs into orbits below the inner Van Allen belt are planned. Each platform in the constellation will carry two sets of electro-optic systems. The first is a high-resolution, wide

field-of-view (FOV) acquisition sensor, which scans from horizon to horizon (plus a few degrees above the horizon to allow full coverage of up to ~30 km above the surface of the Earth) to search, detect, and track missiles during their boost phase. Thus, the sensor operates in the short wavelength infrared band to enable detection of the high-temperature plume of the rockets. When a target is detected, the acquisition sensor performs a handoff to the second system mounted on each spacecraft, called the tracking sensor, to continue monitoring of the missiles' trajectories through midcourse and reentry. The tracking sensor is a slew-and-stare multispectral infrared system (ranging from visible to long wavelength IR) with a narrow FOV, but a large field of regard, resulting in a high agility requirement.

The application example selected as the demonstration problem in this research is to design and deploy a system similar to the eventual operational LEO constellation to be developed through the STSS program. The overall objective is to find the constellation configuration (number of orbital planes, number of satellites per plane, and their relative spacing), the mission orbit (altitude and inclination), and the spacecraft design (sensor size and spacecraft mass) that results in the minimum cost through deployment, consisting of research, design, testing and evaluation (RDT&E), spacecraft production, and launch costs. This multisatellite system is to provide a global continuous single-fold coverage of the Earth, that is, at least one satellite would be visible from all points of the Earth at all times. For this detection system, the spatial (ground) resolution requirements are fixed at 0.4 and 0.3 km for the acquisition and tracking sensors, respectively. The acquisition sensors are required to provide aggregately a continuous global single-fold coverage up to 30 km above the horizon with their cross-track whiskbroom imaging mode. Sensitivity is measured by the signal-to-noise ratio (SNR). For this type of mission, SNR of at least 10 dB is needed.

There are numerous ways to configure the satellites in a constellation system. The Walker delta pattern,<sup>20</sup> which symmetrically places all of the spacecraft in circular orbits at the same altitude and inclination with equal number of satellites in each orbital plane, is commonly used as a starting point in constellation designs.<sup>11,21</sup> The Walker delta pattern<sup>20</sup> offers a variety of configurations that can efficiently provide the coverage required by the application example of this study. Therefore, it was considered reasonable to limit the problem to only this type of constellation.

The mission called for one on-orbit spare satellite to be placed in each orbital plane. For simplicity, it was assumed that the launch vehicles were not capable of multiple-plane insertions on a single flight. The ability to inject a spacecraft into an orbital plane other than their final operational plane was, therefore, not considered.

### Problem Formulation

Three original discipline-level analysis programs with local optimization options were created in support of this research study. The new disciplinary analysis modules correspond to the first three boxes shown in Fig. 1. These modules are briefly discussed hereafter. A more complete treatment of the operation, numerical precision, and key assumptions of these analysis module tools can be found in Ref. 22. Note that a cost module is included to complete the four-module design structure matrix in Fig. 2. The cost module is not an analysis code per se but rather a simple summation of the contributing cost elements. That is, the cost module has no local variables or constraints.

The implementation of the collaborative architecture for this constellation design problem is summarized in Table 2. Interdisciplinary variables, local variables, local constraints, and the disciplinary objective function are identified for each of the three disciplinary modules, as well as for the system-level optimization problem. The various optimization schemes used both at the system and the subspace levels are also listed.

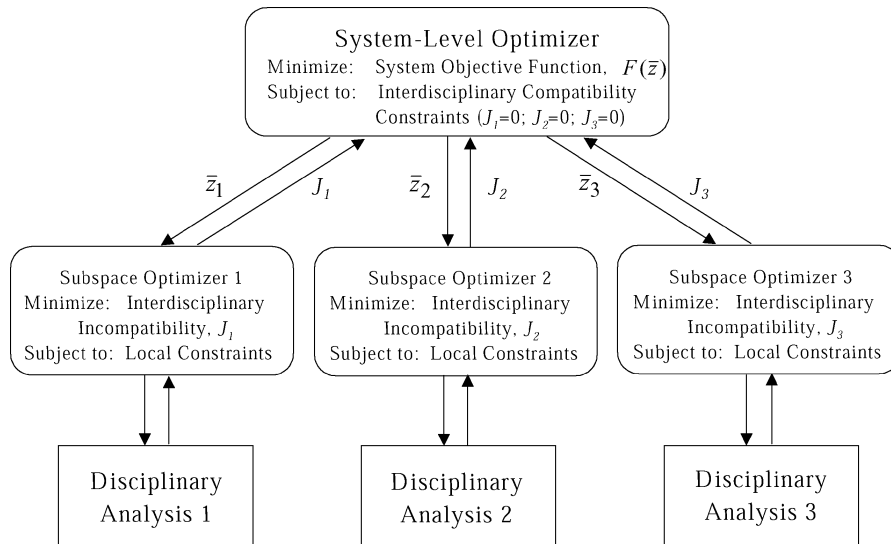
### Configuration and Orbit Design

The configuration and orbit design module performs coverage analysis for different orbit parameters and constellation patterns by propagating the satellites in the constellation and the grid points

**Table 2** Summary of CO's implementation to the constellation design problem

Variable	Constraint	Objective function	Optimization scheme
<i>Configuration and orbit design</i>			
Number of planes	Global continuous	$J_1$	Grid search and heuristics
Number of satellites/plane	one-fold coverage		
Altitude	Closest approach		
Inclination	distance > 150 km		
Minimum elevation			
Relative spacing <sup>a</sup>			
<i>Spacecraft design</i>			
Altitude	SNR > 10 dB	$J_2$	Univariate search and gradient-based search
Minimum elevation	Angular blur <		
Aperture diameters <sup>a</sup>	required		
Pixel size <sup>a</sup>	resolution		
Spacecraft RDT&E cost	Dwell times >		
Spacecraft TFU cost	$3\tau_{\text{det}}$ (detector's time constant)		
<i>Launch manifest</i>			
Number of planes	Launch vehicle	$J_3$	Integer programming and heuristics and univariate search
Number of satellites/plane	availability		
Altitude			
Inclination			
Spacecraft unit mass			
Total launch cost			
Flight rates <sup>a</sup>			
<i>System design</i>			
Number of planes	$J_1 = 0$	Total cost through deployment $F$	Penalty function using Powell's method with quadratic approximation
Number of satellites/plane	$J_2 = 0$		
Altitude	$J_3 = 0$		
Inclination			
Minimum elevation			
Spacecraft unit mass			
Total launch cost			
Spacecraft RDT&E cost			
Spacecraft TFU cost			

<sup>a</sup>Elements of the local variable vector  $\psi_i$ .

**Fig. 2** CO architecture.

distributed over the Earth's surface. It checks at every time step that each grid point lies within at least one satellite's beam width. The coverage analysis program treats each spacecraft as a point mass orbiting a spherical Earth. Effects due to atmospheric drag and solar radiation pressure were ignored, under the assumption of station-keeping capability on each spacecraft to maintain the design orbit. Third-body perturbations due to the moon and the sun and the geopotential effects of the Earth's oblateness were not included

in this analysis because they result in similar nodal regressions and apsidal rotations on all of the satellites in a Walker constellation, such that the pattern (or relative positioning of the satellites to one another) changed negligibly.

With the CO implementation, the configuration and orbit design subspace optimizer is given control over local versions of the coupling variables, forming the vector  $y_1$ . Thus,  $y_1$  consists of the number of orbital planes, the number of satellites per plane, the altitude

of the circular orbits, the inclination of the planes, and the minimum elevation angle, which defines the acquisition sensor beam size. The local variable vector  $\psi_1$  only has one integer component: the relative spacing for the Walker configuration. The system optimizer provides normalized target values  $z_1$  for  $y_1$ , forming the subspace objective function  $J_1$ . Thus, the task of the subspace optimizer is to minimize the differences between the target and the local values by varying the local variables  $y_1$  and  $\psi_1$ ,

$$\min_{y_1, \psi_1} \{J_1\} = \min_{y_1, \psi_1} \{\|y_1 - z_1\|^2\} \quad (1)$$

while ensuring that the local constraints (continuous single-fold global coverage and minimum approach distance between the satellites in the constellation) are satisfied. Because of the nonlinearity of the subspace and the abundance of discrete variables, a combination of exhaustive grid search and heuristics was implemented to optimize this subspace problem efficiently and reliably. Given the rapid execution speed of this analysis module, the use of exhaustive grid search guided by experienced-based rules, which can quickly eliminate inefficient or infeasible options, did not result in a significant time penalty relative to stochastic methods, for example, genetic algorithms, and was guaranteed to locate a global minimum.

### Spacecraft Design

The spacecraft design module estimates the mass and cost of the sensor payload and the supporting spacecraft bus that satisfy the detection performance requirements at the specified mission orbit. First, the aperture diameter and the focal length of both sensor systems were computed based on the spacecraft altitude and the ground resolution and sensitivity requirements. The mass and power requirements of the acquisition sensor could then be determined based on its dimensions by the use of the design estimating relationships found in Ref. 8. The mass and power calculations for the tracking sensor were based on the procedures found in Ref. 23. Finally, the mass of the spacecraft bus required to support the payload was computed from a curve-fit function relating the mass of the bus to the mass and power requirements of the payload, which was developed based on similar existing systems such as NOAA 11 and ERS 2 (Ref. 24). The RDT&E and theoretical first unit (TFU) cost calculations for the payload and spacecraft bus were based on estimating relationships published by Wong.<sup>25</sup>

Within the CO architecture, the spacecraft design module is given the freedom to change local versions of the interdisciplinary coupling variables that are inputs to the discipline tool: orbit altitude and the minimum elevation angle. Furthermore, the spacecraft design subspace optimizer is allowed to vary its local variables, that is, pixel size and aperture diameters of the sensors, which form the vector  $\psi_2$ . As these inputs are changed, the local constraints are recalculated and the resultant cost parameters of the spacecraft are generated. Target values for altitude, elevation angle (related to sensor beam size), spacecraft unit mass, and cost parameters, represented as the  $n_2$ -dimensional vector  $z_2 \ni z_2 \in \mathbb{R}^{n_2}$ , where  $z_2 \subset z$ , are received from the system level. Local versions for these variables are created that comprise the single vector  $y_2$ . Again, the subspace optimizer's task is to meet the target values provided by the system optimizer as best as possible, by varying the local variables  $y_2 \cup \psi_2$ , while satisfying the local constraints listed in Table 2:

$$\min_{y_2, \psi_2} \{J_2\} = \min_{y_2, \psi_2} \{\|y_2 - z_2\|^2\} \quad (2)$$

Although all of the local variables are continuous, the selected gradient-based optimizer that uses a standard sequential quadratic programming (SQP) algorithm had difficulties dealing with the various constraints and finding a solution in a robust manner. A hybrid optimization scheme, first involving a univariate (one-variable-at-a-time) search and then the gradient-based optimization technique, was developed and was found capable of efficiently solving the spacecraft design optimization subproblem. In this approach, altitude and minimum elevation angle were each increased or decreased univariately. At each candidate solution for altitude and minimum

elevation angle, a gradient-based optimization solved for the optimal aperture diameters and pixel size that minimized  $J_2$ , while still meeting the local constraints. Altitude and/or minimum elevation angle were then updated by the use of univariate quadratic approximations, and the process was repeated until the minimum overall  $J_2$  was found that still resulted in a feasible problem.

### Launch Manifest

The launch manifest module finds the minimum launch cost strategy to deploy the given constellation system to the specified orbit. A database was developed to contain the payload capacities of various launch vehicles to a range of altitudes and inclinations and that were launched from different sites. The database is capable of linear interpolation between these data points. Launch costs for the various vehicles were averaged from various publications.<sup>26,27</sup> Also, a limit of no more than eight of each type of launcher was assumed to be available to deploy the constellation within the given deployment schedule. For a given constellation configuration, mission orbit, and spacecraft unit mass, therefore, the launch manifest problem can be formulated as an integer programming (IP) problem because the objective function and the constraints are linear with respect to the discrete variables, comprised of the launch rates of the various launch vehicles needed to populate the orbital planes of the constellation.

Within the CO architecture, however, the variable vector  $y_3$  for the subspace optimizer includes the local versions of the discipline input values: spacecraft unit mass, the mission orbit (altitude and inclination), the constellation configuration (number of planes and number of satellites per plane), and the discipline output variable: total launch cost. The launch rates for each type of launcher form the local variable vector  $\psi_3$ . The launch manifest problem becomes a minimization of the discrepancies between the target and the local values for these interdisciplinary coupling variables, subject to each launch vehicle's availability:

$$\min_{y_3, \psi_3} \{J_3\} = \min_{y_3, \psi_3} \{\|y_3 - z_3\|^2\} \quad (3)$$

This CO reformulation caused the subspace optimization problem to be complex and nonlinear and to involve both discrete and continuous variables. An exhaustive grid search approach was considered too time consuming, given the large number of variables, resulting in a sizeable combinatorial problem. Instead, a three-step process was developed. First, as a starting point, the original IP problem was solved for the minimum cost strategy for deploying the constellation specified by the system-level targets  $z_3$ . Next, expert-based heuristics were used to select an option for improving  $J_3$  quickly through various tradeoffs between the variable components of  $y_3$ , for example, changing the number of orbital planes vs changing the mission orbit to better match the resulting total launch cost to the target value given by the system-level optimizer. Finally, an univariate search performed a tradeoff between the mission orbit and spacecraft unit mass to further reduce  $J_3$ .

### System Design

At the system level, an optimizer coordinates the overall process by selecting target values for all of the interdisciplinary variables  $z$ . Additionally, the system optimizer is required to satisfy the compatibility constraints, such that  $J_1$ ,  $J_2$ , and  $J_3$  equal zero in the final solution. The overall objective function,  $F(z)$ , is the total financial cost through deployment for the constellation system.

The system-level optimization problem is nonlinear and involves a mix of continuous and discrete variables. As such, traditional direct nonlinear optimization algorithms such as SQP could not be applied to this problem. Several methods have been proposed for solving this type of problem,<sup>28</sup> including the modified interior penalty function of Gisvold and Moe.<sup>29</sup> This approach was implemented for this research and was found to work satisfactorily, albeit with some penalty in execution speed. The modified interior penalty function method treats the discrete decision parameters as continuous and attaches a penalty term if these variables hold noninteger values. Penalty

terms were also constructed to enforce the  $J_i$  equality constraints indirectly. The transformed problem becomes

$$\min_z \{\Phi_k(\mathbf{z})\} = \min_z \left\{ F(\mathbf{z}) + r_k \sum_i^3 J_i(\mathbf{z}) + s_k Q_k(\mathbf{z}^d) \right\} \quad (4)$$

With this technique,  $\Phi$  was sequentially minimized over several top-level iterations indicated by  $k$ . The discretization penalty term  $Q_k(\mathbf{z}^d)$ , where  $\mathbf{z}^d$  was composed of the discrete members of the system variable vector  $\mathbf{z}$ , was the normalized symmetrical beta-function integrand suggested in Ref. 29. Because the new system-level formulation resulted in an unconstrained problem to minimize the pseudo-objective  $\Phi$ , Powell's method (see Ref. 28), with three-point quadratic approximation along each line search, was employed at each iteration  $k$ . Within each of these top-level iterations,  $r_k$  and  $s_k$ , the multipliers for the penalty terms, were held constant until convergence was reached. The process continued by incrementation of  $k$ , which initiated a new top-level iteration, and through an increase of  $r_k$  and  $s_k$  to penalize more heavily for violated constraints, that is, nonzero  $J_i$  and noninteger values for  $\mathbf{z}^d$ , respectively, until convergence of the overall optimization problem was met.

### Optimization Results

Several convergence criteria were utilized throughout this constellation design problem. At both the system and subsystem level, convergence was based on comparisons of the absolute changes of the objective functions and of the absolute changes of the variable values between successive iterations. A tolerance of  $10^{-5}$  was applied on each of these criteria, which were computed with normalized values.

At the start of the optimization process, the initial values for the system-level variables were chosen randomly from the range of interest. Successful convergence was achieved after nine top-level iterations. Given the wide range of values explored during the optimization process (Figs. 3–8), one can be assured that this methodology is robust to the initial conditions.

The coordination for the system variables within the collaborative architecture is shown in Figs. 3–8. The behavior of the system-level variables and the local versions controlled by the subspace optimizers revealed the dynamics that made this problem both interesting and challenging. The configuration and orbit design module always preferred larger constellations (more planes and greater number of satellites per plane), higher altitude, and lower minimum elevation angle. The spacecraft design, to come as close as possible to the low-cost targets set by the system optimizer, would bid for lower altitude and increased minimum elevation angle (equivalent to smaller sensor beam size and, therefore, smaller and cheaper spacecraft). Finally, the launch manifest module favored fewer planes to keep launch cost down.

Typical of these dynamics between the system design and the three disciplines, the objective function  $F(\mathbf{z})$  plot in Fig. 9 shows a

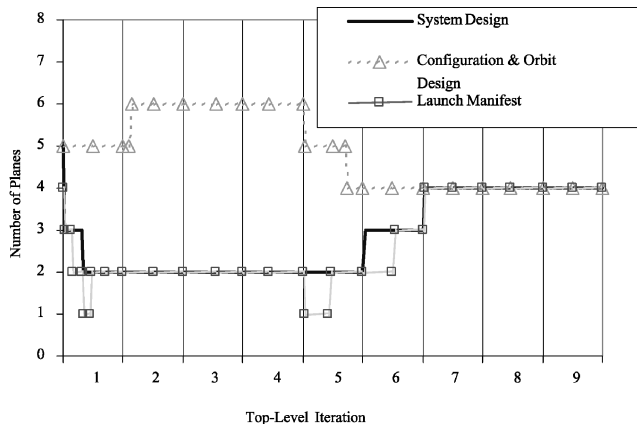


Fig. 3 Progression of number of planes within the collaborative architecture.

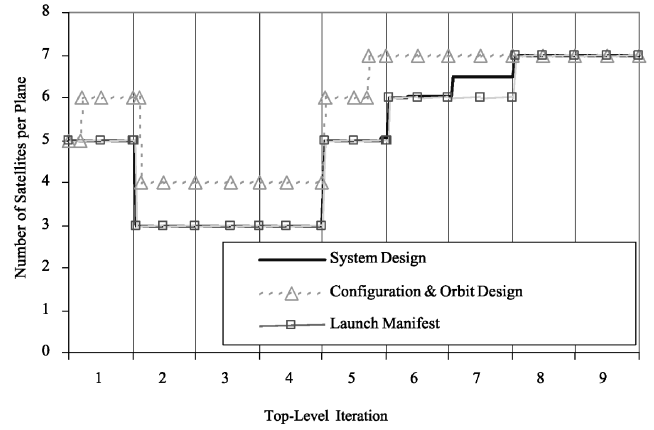


Fig. 4 Progression of number of spacecraft per plane within the collaborative architecture.

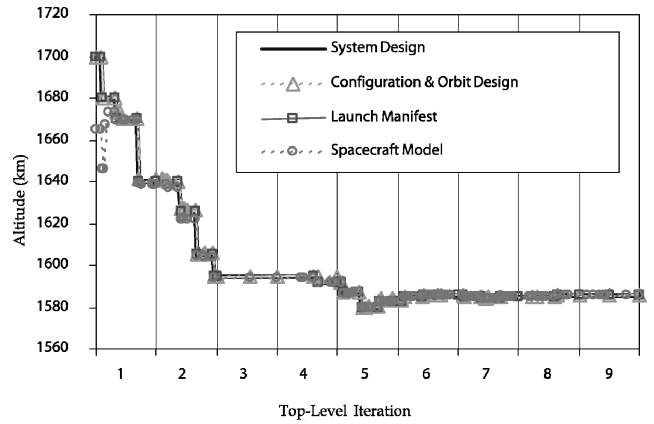


Fig. 5 Progression of altitude within the collaborative architecture.

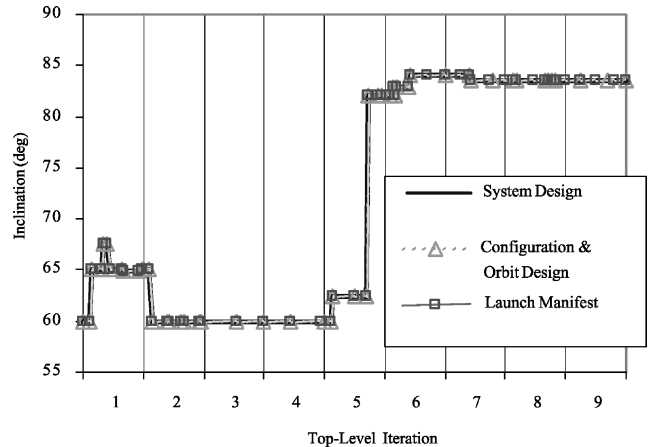


Fig. 6 Progression of inclination within the collaborative architecture.

relatively low cost in iterations 2–6. These were actually infeasible solutions because the number of planes had not converged between the configuration and orbit design module and the launch manifest module (Fig. 3). The configuration and orbit design module preferred more orbital planes for its coverage requirements, whereas the launch manifest module (and the system design target) initially preferred a fewer number of planes to meet the low launch cost target. This discrepancy was eventually brought to zero, albeit at the expense of a higher financial cost  $F(\mathbf{z})$  because the penalty multiplier term  $r_k$  was increased in the final few iterations. Similar trends can be seen in Figs. 4 and 7, which show a reduction in discrepancies between the coupling variables as the penalty multiplier term is increased. On the contrary, inclination and spacecraft unit mass

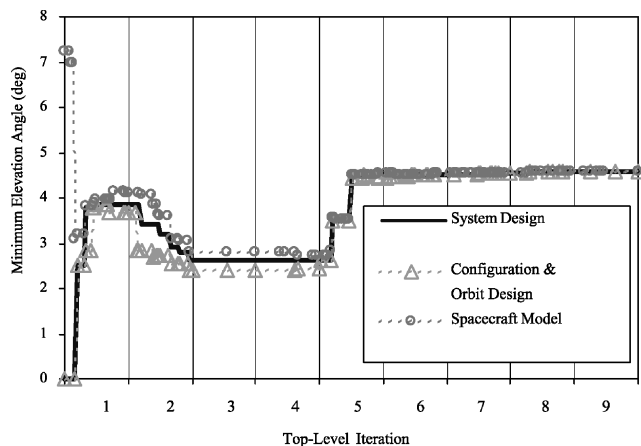


Fig. 7 Progression of minimum elevation angle for acquisition sensor (AS) within the collaborative architecture.

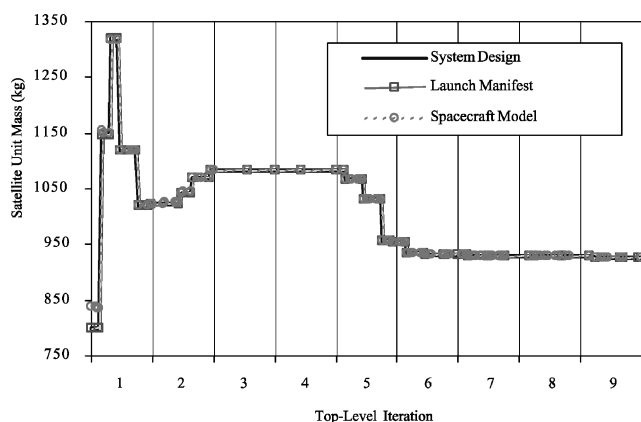


Fig. 8 Progression of spacecraft unit mass within the collaborative architecture.

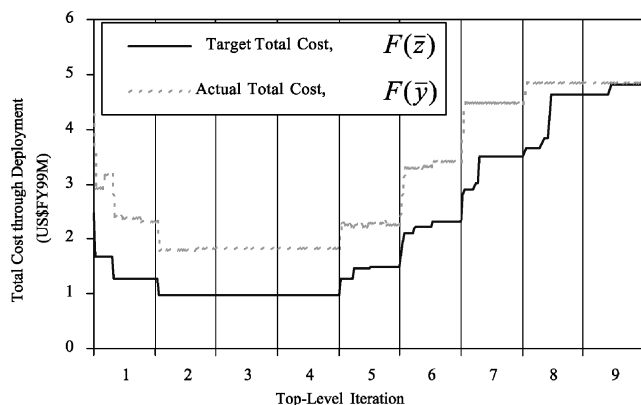


Fig. 9 Progression of the total cost through deployment within the collaborative architecture.

were found to be easily adaptable variables with the subsystems values, varying only slightly from the systems design targets at each iteration (Figs. 6 and 8, respectively).

The description of the resulting optimized constellation is given in Table 3. The 28/4/3 Walker pattern at an orbital altitude of 1585 km and 83.6-deg inclination was the chosen configuration that minimized the total cost through deployment phase. The launch strategy involved one Atlas IIIA and one Delta 7920 to populate each of the four planes, resulting in a total launch cost of \$580 million U.S. dollars, fiscal year 1999 (US\$FY99). An Atlas IIIA is capable of carrying six satellites weighing 930 kg each to the determined orbit. The Delta 7920 can deploy two more, for a total of eight spacecraft per plane (including the one required spare).

Table 3 Summary of the optimized result for the constellation design problem

Variable	Optimized value
Number of planes	4
Number of satellites/plane	7
Relative spacing	3
Orbit altitude, km	1585
Orbit inclination, deg	83.6
Spacecraft unit mass, kg	930
Acquisition sensor nadir angle, deg	53.33
Acquisition sensor minimum elevation angle, deg	4.57
Total launch cost (US\$FY99M)	580
Spacecraft RDT and E (US\$FY99M)	440.6
Spacecraft TFU (US\$FY99M)	164.9
Total production cost (US\$FY99M)	3786.4
Average unit production cost (US\$FY99M)	118.3
Total cost through deployment (US\$FY99M)	4807

The final predicted cost of development, production, and initial deployments was determined to be \$4.807 billion (US\$FY99) by the use of the design modules created for this study. Comparison data for a current STSS configuration are not publicly available at this time. The results of this study are deemed, within reason, to be accurate, given the resources and databases available to the authors, but readers are reminded that the intent of this exercise was primarily to evaluate the performance of CO for creating a distributed, parallel multidisciplinary architecture for multilevel optimization, not to determine physical variable values precisely for the test application that would be subject to additional constraints not modeled in this work.

## Conclusions

With recognition of the need for a systematic, multivariable, multidisciplinary method, the research presented in this article applied CO as a new approach for the system integration and optimization of a satellite constellation. The study involved the design of the constellation's orbit, configuration, individual spacecraft, and deployment strategy. Successful convergence as applied to the design and deployment problem of a space-based infrared constellation system for ballistic missile defense verified the feasibility of CO for solving this type of problem. Although some limitations of CO have been noted, no alternate approaches were quantitatively evaluated for this study.

Several organizational and computational benefits of the CO architecture were demonstrated. First, the method comprised a distributed, parallel architecture that would fit well into today's MDOs such that subsystem flexibility was preserved. With CO, the subsystems retained control of their analysis methods and maintained the freedom to select their computing platforms and optimization algorithms. The collaborative environment provided a second organizational advantage in giving the disciplines some influence on the overall system optimization. Local variables were controlled by the individual subsystems, which allowed them some design freedom. Each discipline also guided the system optimizer through the negotiation processes to meet the local constraints. This decomposition of the system problem further provided scalability to the approach, slowing the growth of problem complexity. Finally, the extensive systematic exploration of the multivariable design space by the chosen method increased the confidence on the goodness of the solution achieved.

Beyond a demonstration of the applicability of CO to satellite constellation designs, this study also extended the current state of CO. First, successful convergence of all the test cases proved the feasibility of the use of nongradient-based optimization techniques within the CO architecture at both system and subsystem levels. Second, this research had also extended the domain of applicability of CO to mixed-integer nonlinear multidisciplinary design problems by demonstrating stability in the system optimization process when an indirect (penalty function) formulation of the system-level objective and constraints is used, leading to a successful convergence.

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